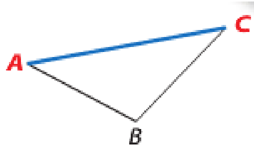
**1 ASA Postulate** An **included side** is the side located between two consecutive angles of a polygon. In $\triangle ABC$ at the right, $\overline{AC}$ is the included side between $\angle A$ and $\angle C$.

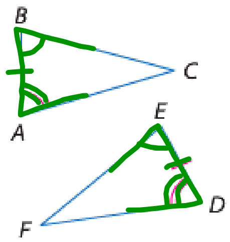
(shared side)




**Postulate 4.3 Angle-Side-Angle (ASA) Congruence**

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

**Example** If **A**ngle $\angle A \cong \angle D$,
**S**ide $\overline{AB} \cong \overline{DE}$, and
**A**ngle $\angle B \cong \angle E$,
then $\triangle ABC \cong \triangle DEF$. by ASA



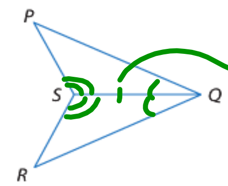
Example 1: Write a two-column proof:

imply $\angle PQS \cong \angle RQS$

**Write a two-column proof.**

**Given:** $\overline{QS}$ bisects $\angle PQR$;
$\angle PSQ \cong \angle RSQ$.

**Prove:** $\triangle PQS \cong \triangle RQS$




$\overline{SQ} \cong \overline{SQ}$

$\overline{SQ}$ corresponds to itself → $\cong$ to itself.

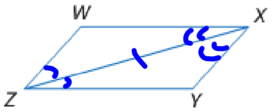
| STATEMENTS | REASONS |
|---|---|
| 1. $\overline{QS}$ bisects $\angle PQR$, $\angle PSQ \cong \angle RSQ$ | 1. Given |
| 2. $\angle PQS \cong \angle RQS$ | 2. Def. of angle bisector |
| 3. $\overline{SQ} \cong \overline{SQ}$ | 3. Reflexive Property |
| 4. $\triangle PQS \cong \triangle RQS$ | 4. ASA |

Example 2: Write a two-column proof:

implies $\angle WZX \cong \angle YZX$

**Given:** $\overline{ZX}$ bisects $\angle WZY$; $\overline{XZ}$ bisects $\angle YXW$.

**Prove:** $\triangle WXZ \cong \triangle XZY$




$\overline{ZX} \cong \overline{ZX}$

implies $\angle WXZ \cong \angle YXZ$

| STATEMENTS | REASONS |
|---|---|
| 1. $\overline{ZX}$ bisects $\angle WZY$, $\overline{XZ}$ bisects $\angle YXW$ | 1. Given |
| 2. $\angle WZX \cong \angle YZX$, $\angle WXZ \cong \angle YXZ$ | 2. Def. of Angle Bisector |
| 3. $\overline{ZX} \cong \overline{ZX}$ | 3. Reflexive Property |
| 4. $\triangle WXZ \cong \triangle XZY$ | 4. ASA |

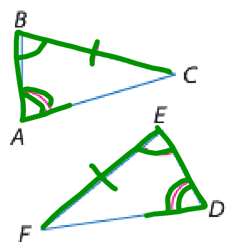
**2 AAS Theorem** The congruence of two angles and a nonincluded side are also sufficient to prove two triangles congruent. This congruence relationship is a theorem because it can be proved using the Third Angles Theorem.

**Theorem 4.5 Angle-Angle-Side (AAS) Congruence**

Not in-between

If two angles and the nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

**Example** If Angle $\angle A \cong \angle D$,
Angle $\angle B \cong \angle E$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.

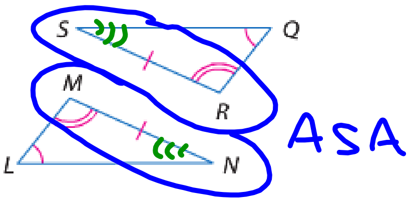


Why does AAS work? Observe the following proof:

**Given:** $\angle L \cong \angle Q$, $\angle M \cong \angle R$, $\overline{MN} \cong \overline{RS}$ → Given AAS

**Prove:** $\triangle LMN \cong \triangle QRS$

**Proof:**




| STATEMENTS | REASONS |
|---|---|
| 1. $\angle L \cong \angle Q$, $\angle M \cong \angle R$, $\overline{MN} \cong \overline{RS}$ | 1. GIVEN |
| 2. $\angle N \cong \angle S$ | 2. THIRD ANGLES THEOREM |
| 3. $\Delta LMN \cong \Delta QRS$ | 3. ASA |

So, this will happen any time we are given two angles with a non-included side congruent to the corresponding two angles and non-included side. Therefore, Angle-Angle-Side is a congruence theorem (but remember, SSA and AAA are NOT!)

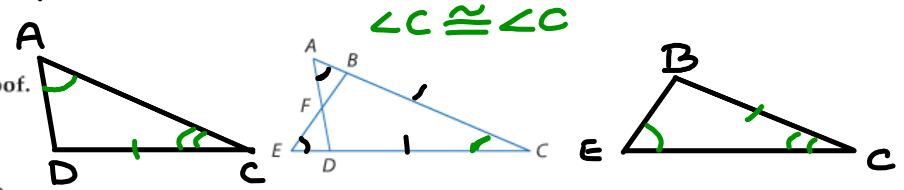
Example 3:

**Write a two-column proof.**

**Given:** $\angle DAC \cong \angle BEC$
$\overline{DC} \cong \overline{BC}$

**Prove:** $\triangle ACD \cong \triangle ECB$

$\angle C \cong \angle C$




overlapping $\Delta$s

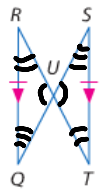
| STATEMENTS | REASONS *separate the $\Delta$s* |
|---|---|
| 1. $\angle DAC \cong \angle BEC$, $\overline{DC} \cong \overline{BC}$ | 1. Given |
| 2. $\angle C \cong \angle C$ | 2. Reflexive Property |
| 3. $\Delta ACD \cong \Delta ECB$ | 3. AAS |
| | |

hint: || lines → ≅ ∠s

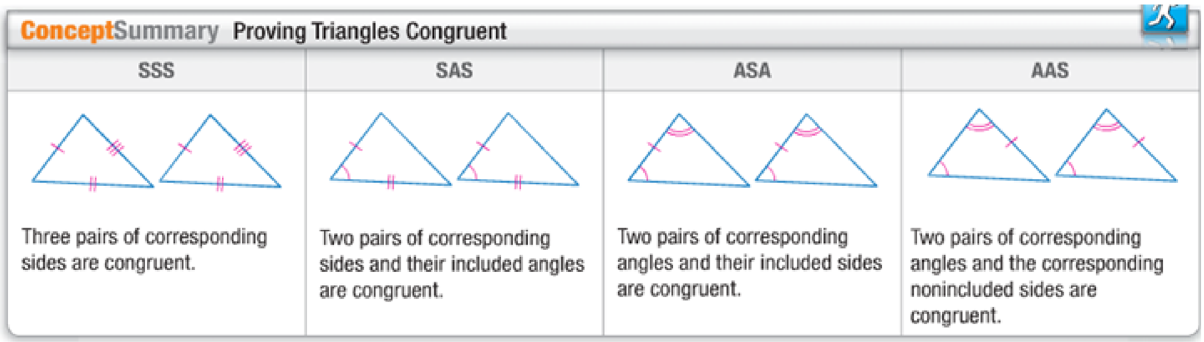
Example 4: Write a two-column proof.

**Given:** $\overline{RQ} \cong \overline{ST}$ and $\overline{RQ} \parallel \overline{ST}$

**Prove:** $\triangle RUQ \cong \triangle TUS$




| STATEMENTS | REASONS |
| --- | --- |
| 1. $\overline{RQ} \cong \overline{ST}$, $\overline{RQ} \parallel \overline{ST}$ | 1. Given |
| 2. $\angle R \cong \angle T$, $\angle Q \cong \angle S$ | 2. Alternate Interior Angles Thm. |
| 3. $\triangle RUQ \cong \triangle TUS$ | 3. ASA |
| 3. $\angle RUQ \cong \angle SUT$ | 3. Vertical Angles Thm. |
| 4. $\triangle RUQ \cong \triangle TUS$ | 4. AAS |



**ConceptSummary** Proving Triangles Congruent

| SSS | SAS | ASA | AAS |
| --- | --- | --- | --- |
| Three pairs of corresponding sides are congruent. | Two pairs of corresponding sides and their included angles are congruent. | Two pairs of corresponding angles and their included sides are congruent. | Two pairs of corresponding angles and the corresponding nonincluded sides are congruent. |